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K_5 with center at F . Let G be one of the points of intersection¹ of K_4 and K_5 . Draw the line $l_4 = GE$ intersecting¹ the line l_3 at the point H . With H as center and radius HA draw the circle K_6 ; let J be one of its intersections with the line l_4 . With AJ as radius and P as center draw the circle K_7 intersecting³ the circle K in the points B and B_1 . Draw the chords $l_5 = BPB'$ and $l_6 = B_1PB'_1$. These are the required chords. Our construction has required the drawing of six lines $l_1, l_2, l_3, l_4, l_5, l_6$ and seven circles $K_1, K_2, K_3, K_4, K_5, K_6, K_7$, the locating of ten necessary points $A, B, B_1, D', D, E, F, G, H, J$. (If one assumes that the center C is not given, as one might from the statement of the problem, the construction is much more difficult.)

Also solved by GEORGE AGINS, E. H. CLARKE, H. N. CARLETON, H. H. DOWNING, EMANUEL GOLDFARB, E. D. GRANT, LAURA GUGGENBUHL, R. A. JOHNSON, MARCIA L. LATHAM, E. W. MARTIN, F. V. MORLEY, A. G. MONTGOMERY, H. L. OLSON, W. B. PIERCE, ARTHUR PELLETIER, JOSEPH ROSENBAUM, ELIJAH SWIFT, CHARLES SCHUMAN, L. G. WELD, C. C. YEN, and the PROPOSER.

2754 [1919, 73]. Proposed by J. W. LASLEY, JR., University of North Carolina.

Given $\bar{x} = \tan^{-1} \frac{z}{\sqrt{x^2 + y^2}}$, $\bar{y} = \tan^{-1} \frac{y}{x}$, $\bar{z} = \log \sqrt{x^2 + y^2 + z^2}$, solve for x, y , and z in terms of \bar{x}, \bar{y} , and \bar{z} .

SOLUTION BY E. S. SMITH, University of Cincinnati

From the given equations, we have $z/(\sqrt{x^2 + y^2}) = \tan \bar{x}$ (1), $y/x = \tan \bar{y}$ (2), and

$$x^2 + y^2 + z^2 = e^{2\bar{z}}. \quad (3)$$

Substituting the value of y from (2) in (1), gives

$$z = x \sec \bar{y} \tan \bar{x}. \quad (4)$$

Substituting the values of y and z from (2) and (4) in (3), we have

$$x = \pm e^{\bar{z}} \cos \bar{y} \cos \bar{x}. \quad (5)$$

Hence, from (2) and (5),

$$y = \pm e^{\bar{z}} \sin \bar{y} \cos \bar{x}. \quad (6)$$

From (4) and (6), we have

$$z = \pm e^{\bar{z}} \sin \bar{x}. \quad (7)$$

Hence, the result is $x = \pm e^{\bar{z}} \cos \bar{x} \cos \bar{y}$, $y = \pm e^{\bar{z}} \sin \bar{y} \cos \bar{x}$, and $z = \pm e^{\bar{z}} \sin \bar{x}$.

Also solved by MARCIA L. LATHAM, E. W. MARTIN, H. L. OLSON, GEORGE PAASWELL, ARTHUR PELLETIER, C. H. RICHARDSON, and D. L. STAMY.

2755 [1919, 73]. Proposed by J. L. RILEY, Stephenville, Texas.

Every number whose square is the sum of the squares of two consecutive integers is equal to the sum of the squares of three integers of which two, at least, are consecutive.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

It is well known that the solution of the equation $a^2 + b^2 = c^2$, where a, b, c are relatively prime integers, is given by the formulas, $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$, where m and n points corresponding to G exist. The radius most easily specified for which the existence of G can be proved is the radius AF . It would not do, for example, to say "let us take any radius greater than a half of AF " because that does not specify an exact radius and it would need to be proved that any radius which was used was actually "greater than a half of AF ". The simplest radius to use for the construction and a logical proof is the radius AF .

³ Here we have assumed as in previous cases that there are points of intersection, but in this case there are none unless $AP \leq AJ$. Since AJ is one-third of the required chord BB' , when it exists, it follows that $AP \leq \frac{1}{3}BB'$, which is impossible if AP is greater than one-third of a diameter of K . If $AP \leq \frac{1}{3}$ of a diameter, there always exists a chord which is trisected at P .

are relatively prime. In the case above $a - b = \pm 1$, whence either (1) $(m - n)^2 - 2n^2 = 1$, or (2) $(m + n)^2 - 2m^2 = 1$. The equation (2) may be derived from (1) by interchanging m and n and changing the sign of n . This would not affect the value of c , nor the algebraic work below,¹ hence we need consider only (1).

Writing (1) in the form $(m - n)^2 - 1 = 2n^2$, and factoring we have

$$(3) \quad [m - n - 1][m - n + 1] = 2n^2.$$

Since the two factors differ by 2 and their product is even, each is even, 2 is the H.C.F., and one factor is twice an odd square and the other the square of an even number. Accordingly we have

$$(4) \quad \begin{matrix} m - n - 1 = 2\alpha^2 \\ m - n + 1 = 4\beta^2 \end{matrix} \quad \text{or} \quad (5) \quad \begin{matrix} m - n - 1 = 4\beta^2 \\ m - n + 1 = 2\alpha^2, \end{matrix}$$

where α and β denote integers. From (3), (4) and (5) we find

$$(6) \quad n = 2\alpha\beta, \quad m = \alpha^2 + 2\beta^2 + 2\alpha\beta, \quad \pm 1 = 2\beta^2 - \alpha^2,$$

the upper sign resulting from equations (4): the lower, from (5). Substituting these values in the expression for c , we find

$$c = m^2 + n^2 = (\alpha^2 + 2\beta^2 + 2\alpha\beta)^2 + (2\alpha\beta)^2 = (\alpha^2 + 2\alpha\beta)^2 + (2\alpha\beta + 2\beta^2)^2 + (2\alpha\beta)^2.$$

But $(\alpha^2 + 2\alpha\beta) - (2\alpha\beta + 2\beta^2) = \mp 1$ from (6), so that c is expressed in the desired form.

It is worthy of note that every solution of the equation $\lambda^2 - 2\mu^2 = \pm 1$ leads to a triangle having the given property, and such triangles can be obtained only from such solutions. All these can be found by developing $\sqrt{2}$ into a continued fraction and taking the numerator and denominator of any convergent as λ and μ respectively. In this way we get the following set of solutions:

λ	μ	m	n	a	b	c
1	1	2	1	3	4	$5 = 0^2 + 1^2 + 2^2$
		5	2	21	20	$29 = 2^2 + 3^2 + 4^2$
3	2	12	5	119	120	$169 = 3^2 + 4^2 + 12^2$
		29	12	697	696	$985 = 12^2 + 20^2 + 21^2$
7	5	70	29	4059	4060	$5741 = 20^2 + 21^2 + 70^2$

3]. Proposed by PAUL CAPRON, U. S. Naval Academy.

Given a parallelogram with center O , vertices $PQP'Q'$, mid-points of sides $ABA'B'$ (cyclic order $PAQBP'A'Q'B'$). Let K be any point of OA . Draw KLH parallel to $Q'Q$ cutting AQ at L , and draw BLM , meeting OA produced at M . Draw MH , parallel to PP' , to meet KLH at H , and draw $B'KE$ to meet BL at E . Repeat, changing A, B, P, Q to A', B', P', Q' , respectively, and *vice versa*. Repeat each of the foregoing, changing P, P', B, B' to Q, Q', B', B , respectively, and *vice versa*.

What are the loci of E and H ? Show that EH passes through A' when K is a point of OA , through A when K is a point of OA' . Consider the effect of interchanging the rôles of A and B .

(This construction, as commonly given, is specialized in these particulars: the parallelogram is rectangular, the divisions of OA are equal, and the locus of H is not found.)

I. SOLUTION BY ARTHUR PELLETIER, Montreal, Can.

Let $OB = a$ and $OA' = b$ be the axes of coordinates and, taking $OK = -mb$, we find the following equations of the indicated straight lines:

$$\begin{array}{ll} (1) \quad (KLH) & x/a + y/b = -m, \\ (3) \quad (MH) & x/a - y/b = 1/m, \end{array} \quad \begin{array}{ll} (2) \quad (BLM) & x/a - my/b = 1, \\ (4) \quad (B'KE) & x/a + y/mb = -1. \end{array}$$

By eliminating m from (2) and (4) we find for the equation of the locus of E , $x^2/a^2 + y^2/b^2 = 1$. Hence when K passes from A to O the point E describes the part AB of an ellipse inscribed in the parallelogram, tangent to the sides at A, B, A', B' . The other parts of the ellipse will evidently be obtained by making the changes indicated in the problem. Similarly, we obtain from (1) and (3) the equation $x^2/a^2 - y^2/b^2 = -1$. Hence the locus of H is the part AHC of an hyperbola tangent to PQ at A etc.

¹ The new n would be negative, consequently $\alpha\beta$ in (6) would be negative. Two of the three numbers, the sum of whose squares is c , would be $2\alpha\beta - \alpha^2, 2\alpha\beta - 2\beta^2$ and the difference is again ± 1 .